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UNCONVENTIONAL METHOD FOR ESTIMATION OF GAS-RESERVES RECOVERY FACTOR AND TIME USING RATE DECLINE TRENDS ANALYSIS

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ABSTRACT:
High accuracy evaluation models for predicting gas-reserves, recovery-factor and time were successfully developed, using production rate decline trend analysis. Existing gas data from wells in the Niger Delta geological formations (Agbada, Akata and Benin) were used to generate decline constants 'b' that were subsequently used in predicting yearly production data for any given period. The yearly data obtained were validated using the actual yearly production records of the original data source. The validated yearly data were used to generate evaluation curves. The evaluation models were subsequently worked out from the shape of the generated curves. The models were then used to estimate reserves (cumulative and initially in place) in each of the reservoirs. The values obtained compared favorably with the respective storage tank and the volumetric materials balance equations values. The percentage accuracy ranged from 99.86% and above. The results of this research simplifies complex simulation methods, improves dynamic fluids computational analysis, reduces time in the conventional rate decline analysis and makes it easy to identify dominated flow and rates decline trends. The models are very flexible and can be applied with high accuracy from the reservoir decline stage to abandonment. They are equally used to estimate the remaining reserves based on the time differences between final and production ($\boldsymbol{t_f} - \boldsymbol{t_p}$) and for the establishment of production and economic decisions techniques.

KEYWORDS: Unconventional Gas Reserves Estimation, Rate Decline Trends, Rate Decline Constant, Projectile and Parabolic Methods

INTRODUCTION:

DEFINITION:

Decline Curve Analysis is a procedure to study reserves recovering rates, using production data or history, based on mathematical equations, tabulated values and graphical representation. Or Decline Curve Analysis is a Curve-Fitting & Extrapolation Method Where, Sample curves are matched-up Standard curves generated with regional data. Reserves prediction is by extrapolation of the matched samples curve to desired points**.**

BACKGROUND INFORMATION:

There are no fundamental theoretical trends for decline curves analyses, but the exercise is based on production data trend. For this the principal challenge is to minimize errors. All data must be understood before use. There are three principal types of decline rate as postulated by the early researcher. These are exponential or constant decline rate, harmonic decline rate and hyperbolic decline rate. This classification is based on constant or variable changes in the factors that influence the fluid flow in a porous medium. The equation of a fluid flow through porous media under boundary conditions is based principally on steady-state, semi-steady state and unsteady-state and are applied as deemed fit for any particular situations single or two phase fluid stream. Any stream can exhibit any type of decline rate. It depends on the influencing factors. The practical approach to gas production decline rate analysis is to choose the variable such as gas which results in a reasonable trend. The decline rate trends are used to predict the future well performances. The accuracy in predicting the future gas stream performance depends on the ability to understand the reservoir characteristics and the standard established for estimating the reserves. In this case best rate decline trends analyses would be compared with volumetric calculated values, MBE values and recovery factor values. The decline curves analysis results will be the estimation tools for the cumulative hydrocarbons production and hydrocarbons initially in place. Field records showed that recoverable hydrocarbons are affected by the operating conditions.

When a well is placed on production, there will be transient flow initially, because the boundary conditions are not active enough. Eventually the reservoir boundaries would be felt and it is only then that decline rate becomes clear to predict the value of the decline rate constant (b). **It** is very useful to have production decline rate model in the Niger Delta and other fields in order to predict projected production rates and estimate both reserves in place and the recovery factor in a reservoir. This equally defines the production decline trend and the process that starts a transient state, peak and decline to minimum level or abandonment rate. The decline models would enable a prediction of the recovery efficiency profile, gives the investors much knowledge of his business profile or trend. Many reserves are abandoned early, because of complex simulation procedures in order to establish motivated economic techniques. Conventionally, volumetric material balance equations (MBE) methods in use are limited to static conditions of the reservoirs and less accurate in the dynamic fluids computation analysis. Equally conventional decline analysis is less accurate, because most researchers assumed exponential or constant rates decline. In reality some reservoirs are not. In this work, mathematical equations or relationships are developed to increase DFCA accuracy. To justification this study, it is necessary to simplify the complex simulation procedures in the conventional methods for rate decline analysis. This would increase DFCA accuracy, reduce the simulation complexity and time used. The success of this work will give an investor the view of his business and it improves his decision on the business. This work primarily covers production decline rates characterization for some gas wells in the Niger Delta. The collated data covered the unsteady-stage (early-stage), steady-stage and semi steady-stage (decline-stage) of a reservoir. The complete production data to abandonment can be used for mathematical equations derivations and confirmation.

REVIEW OF THE SIMULATION AND MODELING IN RESERVES ESTIMATION:

Arps, (1945)[1] used an empirical relationship and analyzed hydrocarbons production decline curves. In his work he defined hydrocarbons production decline rate as a factional change (a) in the flow rate (q) with respect to time (t). His mathematical equations are:

$$a = \frac{-{dq}/{dt}}{q_i},\ stb/d \text{ or } stb/yr \quad \text{and} \quad N_p = \frac{q_i - q}{a} \qquad 2.1$$

CRAFTS AND HAWKINS, (1959)[2] field records showed that In decline curve analysis it is implicitly assumed that factors causing the historical decline in a fluid stream would continue unchanged throughout the forecasting period and .these factors are the reservoir and operating conditions. The flow rate was plotted against time to predict projection rates and the daily gas production was plotted against time to estimate future cumulative production and reserves originally in place. The most convenient dependent variable is the rate, because extrapolation of the rate-time graph was used directly to forecast the fluid production and economic evaluations. Plots of rate against daily gas production rate equally provided direct ultimate recovery at a given economic limit and yielded a more rigorous interpretation where the production was influenced by intermittent operations.

Katz, D. L., (1959)[3] "Handbook of Natural Gas Engineering" McGraw-Hill, Inc., New York.

Arps, (1962)[4] used his models in the prediction of oilfields production decline rate types. Here Arps pointed out that there are 3-main types of production decline rate power constants (n). These are the constant or exponential decline rate (where n = 0), hyperbolic decline rate ($where\ 0 < n < 1.0$) and harmonic decline rate (where n = 1.0). He plotted production data against time in a semi-log paper and found out that it gives a straight line graph which could be extrapolated to estimate the oilfield reserves. This was possible, because the drop in production per unit time was a constant fraction of the hydrocarbon production rate.

$$a = -\frac{dq}{q\,dt} = constant \qquad 2.2$$

In the hyperbolic decline rate, he (Arps) found out that the decrease in production per unit time as a fraction of the production rate is proportional to a fractional power. The coefficient of his fraction decline when $0 < n < 1.0$ was given as: $N_p = \frac{q_i}{a(1+n)}\left(q_i^{1-n} - q^{1-n}\right)$ where $q = \frac{q_i}{(1+nat)^{\frac{1}{n}}}$. The coefficient of the decline rate for harmonic decline is unity (n = 1), so the equations become. $q = \frac{q_i}{(1+nat)}$ and $N_p = \frac{q_i}{a} ln \frac{q_i}{q}$ 2.3

EDWARDSON, ET AL (1962)[5] provided the mathematical equation for cumulative hydrocarbons values estimation using dimensionless terms: When $t_D > 200$, $Q_D = \frac{-4.23\, t_D^{0.5} + 2.026\, t_D}{ln\, t_D}$ and when $t_D < 200$, $Q_D = \frac{1.12838t_D^{0.5} + 1.19328t_D^{1} + 0.27t_D^{1.5} + 0.086t_D^{2}}{1 + 0.62t_D^{0.5} + 0.041301t_D^{1}}$ & 2.4

BRUNS, (1986)[6] tried, using fractions as $\frac{1}{2}$, $\frac{5}{8}$ and $\frac{3}{4}$ in his dimensionless time-function and found out that using $\frac{1}{2}$ reduces the discontinuity between the transient streams and hyperbolic streams.

BAILEY, (1982)[7] investigation showed that in some fractured gas wells the rate declined value ''b'' is greater than unity and sometimes as high as 3.5.

FETKOVITCH, (1984)[8]**,** concluded that in commingled layered reservoirs the values of ''n'' lies between 0.5 and 1.0. In such a case decline analysis should be initialized from the start of the decline rate. He added that it is possible under certain production and scenarios that initially the rate does not decline. **Fetkovitch** designed an advanced decline curves analysis approach, which has been applicable for changes in pressure or drainage. His approach was similar to pressure testing. $\frac{q}{q_i}$ VS $a\,t$ or q_{Dd} VS t_{Dd} He also used different values of ''n'', in Arps equations and plotted out curves. From these curves Fetkovitch concluded that Arps' equations are only suitable for rate-time depletion data, but in transient time data will result in incorrect forecasts. In the full size type curves by Fetkovitch field data were plotted on a tracer paper, which are the same as log-log paper scale as the full-size types curves. The best fit in bbl/unit time would be chosen. A match can be used to obtain values of q_i & q for actual data. These data are then used for appropriate equations to be used in the analysis of the rate-time as well as cumulative hydrocarbons production **(N_p or G_p).**

BLASINGAME, ET AL (1989)[9] introduced the concept of integral type curves in the well testing fields. They developed type curves which showed the analysis of transient stems along side with the analytical harmonic decline, but with the rest of the empirical hyperbolic stems absent. Blasingame's hydrocarbons production decline techniques are not limited to constant bottomhole flowing pressure like those in Arps and Fetkovitch. Their hydrocarbons production decline techniques account for variations in bottomhole flowing pressure in the transient regime. In addition their analysis can work fine in the changing values of reservoir PVT properties with the changing reservoir pressure for both oil and gas. They also stated that, if a mechanism maintains the reservoir pressure, the production rate would remain fairly constant. This means that at constant reservoir pressure the decline tends to zero. This is common in pressure maintenance systems, such as gas & water injections, active-water drive, and gas-cap expansion drive, where the hydrocarbons are saturated. Small reservoir pressure decline leads to high production driving force with a corresponding small production decline rate. In this case the decline rate constant is theoretically greater than unity ($n > 1$). Much later when the oil column thins, the production rate would decline exponentially with $n = 0$ and the hydrocarbons production is replaced by water. Advantages in their work were the development of oil and gas production decline method that uses superposition time function that only requires one depletion stem for type curves matching, one of the importance of his method was the type curves used for matching, were identical to those used for Fetkovitch decline analysis without the empirical depletion streams. When the type curves are plotted using Blasingame's superposition time function the analytical exponential stem of Fetkovitch's type curves becomes harmonic. The significance of this is that if the inverse of this flowing pressure is plotted against time, pseudo steady state depletion at constant flow rate follows a harmonic decline. In effect it allows depletion at a constant pressure to appear as pseudo steady state depletion at constant rate, provided that the rate and pressure decline monotonically.

ECONOMIDES, ET AL (1994)[10], considered an oil well drilled in a volumetric oil reservoir where they assumed that the wells production rate starts to decline when a critical (lowest permissible) bottomhole pressure (BHP) is reduced. Under the pseudo-steady-state flow condition the production rate at a given decline time (t) was expressed mathematically as:

$$\mathbf{q} = \frac{k\,h\,(P_t - P_{wf})}{141.2\,B_o\,\mu\ln\left(\frac{0.472\,r_e}{r_w}\right) + S} \quad \textbf{and} \quad \mathbf{N_p} = \int_0^t \frac{k\,h\,(P_t - P_{wf})\,dt}{141.2\,B_o\,\mu\ln\left(\frac{0.472\,r_e}{r_w}\right) + S} : \mathbf{N_p} = \frac{C_t\,N_i}{B_o}(\mathbf{P_0} - \mathbf{P_t}) \qquad 2.18$$

Where: $\mathbf{P_t}$ = Average reservoir pressure at decline time, t and $\mathbf{P_{wf}}$ = **C**ritical BHP during production decline. $\mathbf{N_p}$ = Cumulative production of the well after the decline time (t), C_t = Total reservoir compressibility, N_i = Initial oil in place in the well drainage area and P_0 = Average reservoir pressure at decline time zero.

RAMSAY AND GUERRERO, (2002)[11], Study also included relative decline rate and they indicated in their work that about 40% of leases have $b > 0.5$ and commingled layered reservoirs fall between $0.5 < b < 1.0$.

KING-HUBBERT AND ROBERTSON, (2004)[12], suggested in their work ''Modified Hyperbolic Decline'' that at some point in time the hyperbolic decline is converted into an exponential decline. They extrapolated hyperbolic decline over long periods of time and found out that it frequently results in unrealistically high pressure. To avoid this problem, they made their suggestion. They assumed that for a particular example, the decline rate (D) starts at 30% of flow and declines in a hyperbolic manner. When it reaches a specified value say 10% of the hyperbolic decline it converted to an exponential decline. The error here is that exponential decline rate of 10% would be considered in the forecast. Fig 2.4 shows the graphical representation of their work:

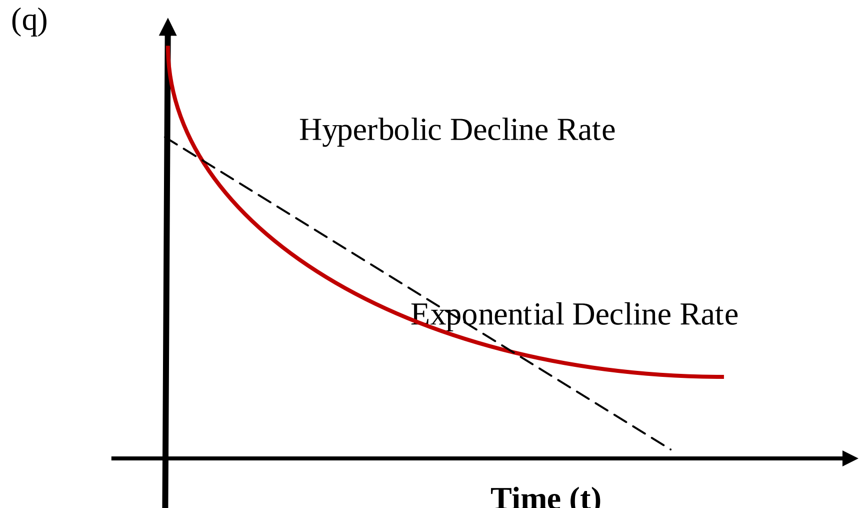


Fig 2.4 Conversion Hyperbolic into Exponential Decline Trend

MATHEMATICALLY:

$$\boldsymbol{q = q_i \frac{[(1-B)^b e^{-Dt}]}{1-B\, e^{(-Dt)^b}}} \quad \text{and} \quad \boldsymbol{Q = q_i \frac{1-B}{BD}\left[1 - \frac{1}{1-B\, e^{(-Dt)^{b-1}}}\right]} \quad 2.23$$

When b = 1: $\boldsymbol{Q = q_i \frac{1-B}{BD} ln\left[1 - \frac{1-Be^{-Dt}}{1-B}\right]}$ Or $\boldsymbol{D = \frac{D_i}{q_i^{b-1}(1+bD_i t)^{\frac{1}{b}}}}$ and $\boldsymbol{q = q_i(1+bD_i t)^{-\frac{1}{b}}}$

Amini, et al, (2007)[13], reservoir model used elliptical flow to govern flow regime in a low permeability gas reservoir with elliptical outer binding. He described these cases as one production from an elliptical wellbore, elliptical fracture or a circular wellbore in an anisotropic reservoir system, which can be considered to be an elliptical inner boundary. They stated that an elliptical reservoir surrounded by an elliptic aquifer is an elliptical outer boundary. They also stated that the reservoir is assumed to be a single-layer system that is isotropic, horizontal and uniform thickness and constant flow rate. Mathematically:

$$\boldsymbol{q_D = \frac{141.2\, B\mu q}{K\, h\, \Delta P}} \quad \text{and} \quad \boldsymbol{K = \frac{141.2\, B\, \mu}{h}\left[\frac{q/_{\Delta P}}{q_D}\right]} \quad 2.20$$

AGARWAL AND GARDNER, (2008)[14], presented new decline type curves for analyzing production data. Their method builds on Fetkovitch's and Palacio-Blasigame's ideas. They utilized the concept of the equivalence between constant rate and constant pressure solution. They also presented new type curves with dimensionless variables based on the conventional well-test definition as in Fetkovitch and Blasigame. They equally included primary and semi-log pressure derivatives plots (decline analysis inverse formant). They as well presented rate versus cumulative and cumulative versus time plots. Rate – cumulative Production analysis mathematically: $\boldsymbol{Q_{DA} = \frac{t_{DA}}{P_D} = q_D t_{DA}}$ **and** $\boldsymbol{q_D = \frac{141.2\, qB\mu}{K\, h\,(P_t - P_{wf})}}$ **and they** explained the importance of water influx in gas reservoir. They observed that an appreciably water influx in a gas reservoir acts as pressure maintenance naturally delaying the decline initiation. The benefit is that much of the hydrocarbons are produced. The disadvantage is that such a reservoir is difficult to model, due to less knowledge of the aquifer behavior and life span.

ILK, ET AL (2008)[15], presented the ''Power - Law'' decline method which uses a different functional form of D-Parameter given by:

$\boldsymbol{D = D_\infty + D_1 t^{-(1-n)}}$ 2.7

D is approximated by a decaying power-law function from transient and through transition flow and exhibits a near constant behavior ($\boldsymbol{ie\ D_\infty}$) at very large time. This is contrast to hyperbolic rate decline that leads to a constant behavior at early time and becomes a unit slope power law decaying function at larger times. The advantage of their mathematical equation is that it is flexible enough to cover the transient, transition and boundary dominated flow and to large time reduces to an exponential decline ($\boldsymbol{D = D_\infty}$). They then combined their equation with Arps' equation as: $\boldsymbol{\frac{1}{D} = \frac{q}{dq/dt} + D = D_\infty + D_1 t^{-(1-n)}}$, Solving gives

$\boldsymbol{q = q_i e^{\left[-D_\infty t - \frac{D_1}{n} t^n\right]}}$ 2.8

When, $D_1 = Decline\ constant,\ t \to \infty, n = Time\ exponent$ and $q_i = Rate\ intercept\ at\ t = 0$. The difference between their $q_i\ and\ q_i$ in Arps decline models is because it refers to rate at the onset of stabilized flow, while q_i in Arps decline models refers to flow rate at early stage of a well.

Obah, et al (2012)[16] used a dynamic simulator and generated a 3D generic grid model with varying oil column thickness, gas-cap and aquifer size. Their based grid was 10 x 10 grid block in the x and y directions. The model geometry was fixed at 600ft x 600ft in the x and y directions, while the z-direction was varied based on the oil rim thickness. They obtained 3-production forecast models for oil rim reservoirs, using Monte Carlo Simulation approach and generated a probabilistic range of forecasts for decision making in the Niger Delta, Nigeria for 30 years. They found out that oil recovery varies from 3.98 – 37.3MMstb over the 30years prediction. They concluded that horizontal wells are better option for developing reservoirs with oil rim as to conventional wells. They also added that oil recovery is strongly dependent on the oil rim thickness, relative gas-cap size (in-factor), permeability, viscosity and aquifer strength. Their mathematical equation was: $\boldsymbol{N_{p(t)} = \frac{q^*}{D}\left(e^{-D/t_p} - e^{-Dt}\right) + q_i t_p}$

REVIEWED EVALUATION AND RESEARCH PROPOSAL:

Evaluating the early researchers' works, it is observed that the whole work is based on identifying exponential, hyperbolic or harmonic decline. They used semi-log fit or cross-match that an exact fit of data was not easily possible. The principal challenges were to improve reserves estimation errors, projecting future reserves production and time required for reserves recovery. The attempt to estimate reserves initially in place and the accuracy in DFCA has not been properly delineated. The gap I intent to fill is to improve reserves (N_p and N) estimation accuracy from 60/67% to 90/99%, reduce the time used in simulation, substituting the exponential, hyperbolic and harmonic decline constants with projectile and parabolic flow decline trends. This is because projectile and parabolic flows make it easy to achieve rate decline trends constant through flow order which had been difficult to achieve.

MATERIALS AND METHODS:

MATERIALS:

The materials used in this work were collected from DPR, NNPC namely, daily operation logging data of oil and gas wells located in the Niger Delta areas. The wells covering Exploration (wildcat) wells, Appraisal (out-step) wells and Production (exploration development) wells. The main data were early to abandonment stages rates. The first set of data were specifically from the exploration, appraisal and production wells, because those wells could define early-stage to the actual production data records, while the second set of data were from the tanks-farms yearly production records (surface facilities) of the same Niger Delta formation oil wells These were used mainly for validating the input data. Table 3.1 shows field data of gas reserves production for $22\frac{1}{2}$ years.

Table 3.1: Field Data for Gas Production in $22\frac{1}{2}$ $Years$

Date	Time, t (yr)	Rate q, MM scf/d
1977	**0**	**0**
1978	**1**	**50**
1979	**2**	**100**
1980	**3**	**100**
1981	**4**	**100**
1982	**5**	**100**
1983	**6**	**100**
1984	**7**	**100**
1985	**8**	**100**
1986	**9**	**100**
1987	**10**	**100**
1988	**11**	**100**
1989	**12**	**100**
1990	**13**	**100**
1991	**14**	**100**
1992	**14.45**	**100**
1993	**15**	**89.60**
1993	**16**	**73.34**
1994	**17**	**60.05**
1995	**18**	**49.16**
1996	**19**	**40.25**
1997	**20**	**32.96**
1998	**21**	**26.98**
1999	**22**	**22.09**
1999	**22.5**	**20.00**

RESEARCH METHODOLOGY:

Raw data for the analysis were collated or grouped into three main dynamic characterizations. Initials to abandonment rates of production, Initials to a given period rates of production and Short period production rates history.

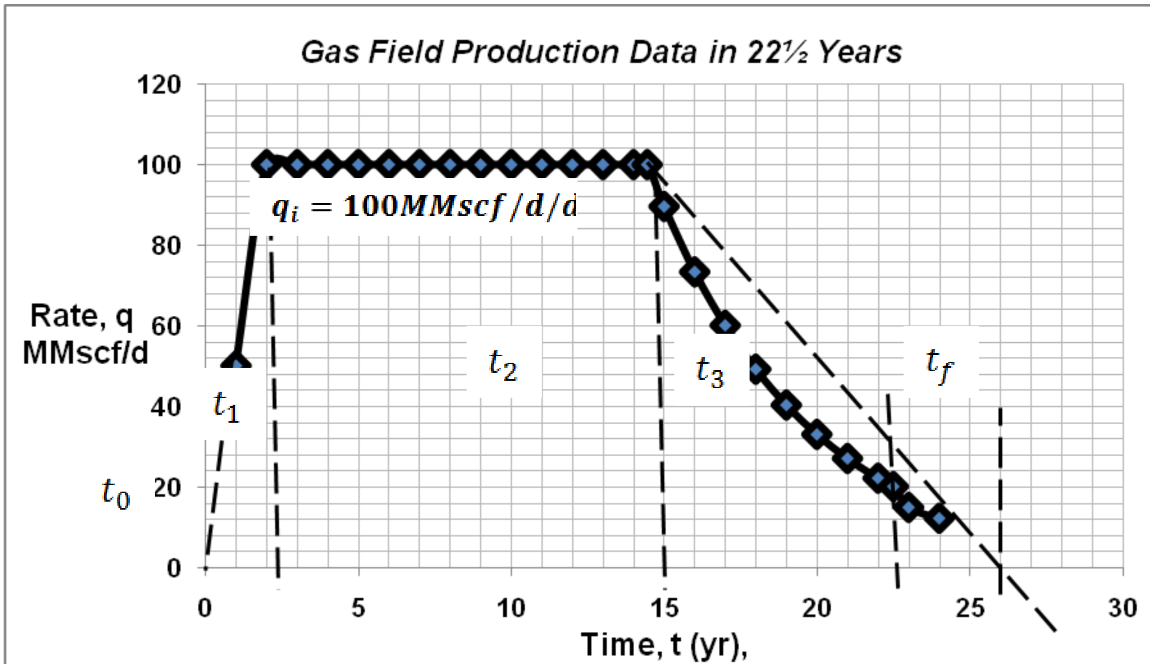
EVALUATION MODELS– I: [GOVERNING MODELS]:

Initial rates to abandonments were plotted against time to generate governing evaluation curves. The curves were used to obtain rates decline constants, **'' b'',** the decline constants, **'' b''** were used to predict yearly rates, the yearly rates were used to build evaluation models and the models were then used to estimate reserves **[G_p & G].**

EVALUATION MODELS – II:

Initial rates to given periods of production were analyzed for decline constant, **''b'',** the decline constant, **''b''** was used to predict yearly rates, the yearly rates were used to generate evaluation curves to the given periods of production & extrapolated the curves to abandonment, the extrapolated curves were used to build evaluation models and the models were then to estimate reserves **[G_p & G].**

Analysis Procedures: Data Type – I



POSTULATION OF THE PROJECTILE GAS FLOW MODELS:
In this section the principal method for postulating the evaluation models was the projectile dominated flow of the reserves. The projectile flow was found common in the depletion of natural gas reservoirs from the initial stage to an abandonment stage. Natural gas reserves recovery values on table 3.1 were plotted against time to generate curves, which were used to study the complete reserves recovery from the initial stage through the transient stage, steady stage, the decline stage to economic rate called abandonment rate (Figure 3.1). The resulted curves in projectile shapes were used to build the models for studying the decline trends and projected to both given recovery periods for estimating the cumulative reserves and zero declined for estimating the reserves initially in place.

CONSIDERATIONS POINTS:
An oilfield must contain a reserve initially in place (N), which reduces per unit time, due to production operation. The flow rate (q) of gas stream production continues to change from time-t_o to time-t_1 and from time-t_1 to time-t_2 and from time-t_2 to time-t_3, (Figure 3.2), so that time-t_f could be extrapolated to the initial reserves values. The gas production (G_p) per unit time declined from the initial value to minimum $\left(\frac{dq}{dt} = -bq^n\right)$. The constant of proportionality is -b. The quantity of the reserves remaining in the reservoir is G_f

CONSTRUCTION:
Joining pt-B to pt-E gives the trapezium ABEO and pt-B to pt-D gives the trapezium ABDO respectively. The general equation for natural production of a gas field reserves is given as Eqn3.1.
Reserves is given as Eqn3.1.

Evaluation Model – 1: The Projectile Oil Flow

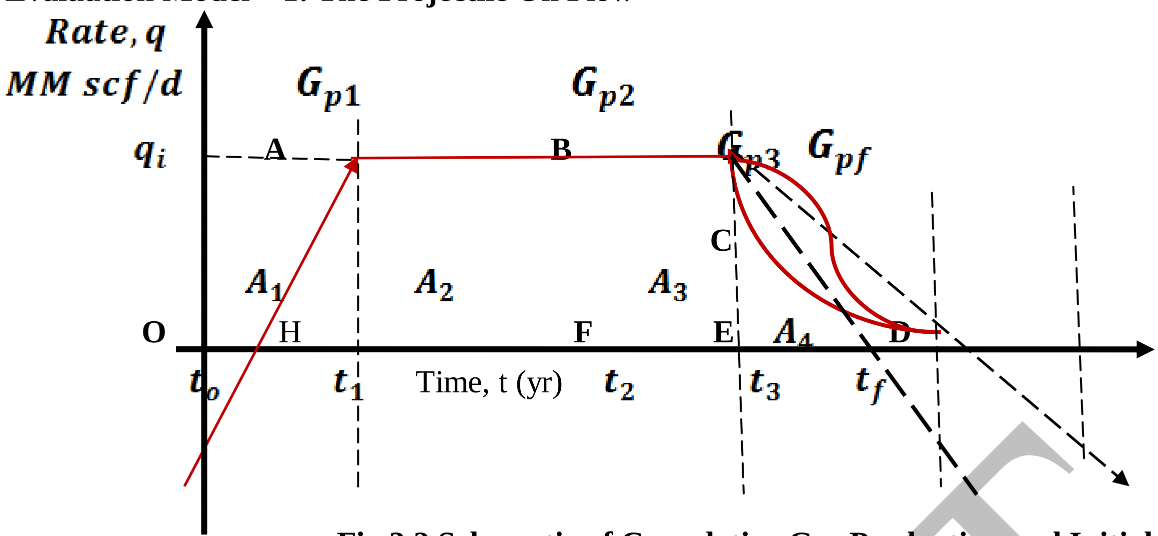


Fig 3.2 Schematic of Cumulative Gas Production and Initial Gas

$$\begin{bmatrix} \text{Actual Reserves} \\ \text{Produced in a} \\ \text{Given Time} \end{bmatrix} = \begin{bmatrix} \text{Actual Reserves} \\ \text{Initially in Place} \end{bmatrix} - \begin{bmatrix} \text{Actual Reserves} \\ \text{remaining in Place} \end{bmatrix}$$

$$G_p = G - G_f \qquad 3.1$$

Using Figure 3.2, the actual gas reserves produced in a given time and gas initially in place are expanded as:

$[Gas\ Produced] = [Area\ of\ the\ Trapezium\ ABEO]$

$[Gas\ Produced] = \frac{1}{2}[Sum\ of\ the\ Parallel\ Sides] * [Height]$

$G_p = \frac{q_i}{2}[(t_2 - t_1) + (t_3 - t_0)]$ or 3.2

$[Gas\ Produced] = [Area\ of\ the\ Trapezium: ABEO] = [A_1 + A_2 + A_3]$

$A_1 = \frac{q_i}{2}[t_1 - t_0] = Area\ of\ \Delta AHO$

$A_2 = q_i[t_2 - t_1] = Area\ of\ the\ rectangle\ ABFH$

$A_3 = \frac{q_i}{2}[t_3 - t_2] = Area\ of\ \Delta BEF$

Substituting A_1, A_2 *and* A_3, gives Eqn3.3. If you so desired, using equation of the curve part ($A_3 = G_{p3} = \frac{q_i}{b}(1 - e^{-bt})$) in Figure 3.2, gives the Oil recovered in Decline rate Stage. This model derived from the first principle below. $G_p = \frac{q_i}{2}[(t_2 - t_1) + (t_3 - t_o)]$ 3.3

YEARLY HYDROCARBONS PRODUCTION PROJECTION:

The general equation for natural production of a gas field reserves is the product of the rate-constant and the actual rate raised to power-n. This is given mathematically by Eqn3.4:

$$\begin{bmatrix} \text{Actual Change} \\ \text{in Production} \\ \text{Rate with Time} \end{bmatrix} = \begin{bmatrix} \text{A Decline Rate} \\ \text{Constant} \end{bmatrix}\begin{bmatrix} \text{Actual Rate in} \\ n - \text{order} \end{bmatrix}$$

$$\frac{dq}{dt} = -bq^n \qquad 3.4$$

Using the curve in Figure 3.2 and integrating Eqn3.4, gives the governing equation, Eqn3.6. The governing equation, Eqn3.6 is used to postulate actual yearly gas production rate (q) by removing the log and rearranging. To estimate the rate-constant (b), the governing equation, Eqn3.6 is rearranged to obtain the rate-constant (b).

Three main flow orders of decline rates were considered.

When n = 1: 1st Order Flow Decline Rate

$$\int_{q_i}^{q} \frac{dq}{q} = -b \int_{o}^{t} dt \qquad 3.5$$

$$\ln q - \ln q_i = -bt : \quad q = q_i e^{-bt} \quad \text{and} \quad b = \frac{\ln(q_i/q)}{t_2 - t_1} \qquad 3.6$$

CUMULATIVE AND GAS -RESERVES RESERVES INITIALLY IN PLACE (G) MODEL POSTULATION

$$\begin{bmatrix} Cumulative\ Gas\ Recovery \\ [Area\ ABEO] \end{bmatrix} = \begin{bmatrix} Area\ of\ the \\ Trapezium\ ABEO \end{bmatrix}$$

$$G_p = [Area\ ABEO] = \frac{q_i}{2}[(t_2 - t_1) + (t_3 - t_o)] \qquad \mathbf{3.7}$$

$$\begin{bmatrix} Actual\ Gas\ Reserves \\ Initially\ in\ Place \end{bmatrix} = \begin{bmatrix} Area\ of\ the \\ Trapezium\ ABDO \end{bmatrix}$$

$$G = [Area\ ABDO] = \frac{q_i}{2}[(t_2 - t_1) + (t_f - t_o)] \qquad 3.8$$

Equation 3.7 is the cumulative gas produced and equation 3.8 is the actual gas oil initially in place (G). This is very possible since gas production is the product of the flow rate, q and time, t (G $= q * t$).

POSTULATION OF THE PARABOLIC OIL FLOW MODELS DATA TYPE – II: CONSIDERATIONS

An oilfield must contain a reserve initially in place (N), which reduces per unit time, during production operations. A reserve must decline right from initial stage during production in a parabolic dome-shape (Figure 3.3a) or single-apex shape (Figure 3.3b). The flow rate (q) of oil stream production continues to change from time, t_o to time, t_1, so that time, t_f could be estimated, by extending Pt-X to Pt-y at time-t_f. The actual change in a production rate per unit time is $dq \propto q^n dt$ and the constant of proportionality is –b or it is the product of the decline rate constant, b and flow rate raised to power-n ($-bq^n$). The cumulative hydrocarbons production (N_p) per unit time would be reduced from the maximum at bubble point (transition state) value to minimum at a given time. The quantity of the reserves remaining in the reservoir is N_f at time-t_f.

PARABOLIC FLOW TYPES – 2, WITH SHORT TRANSITION TIMES

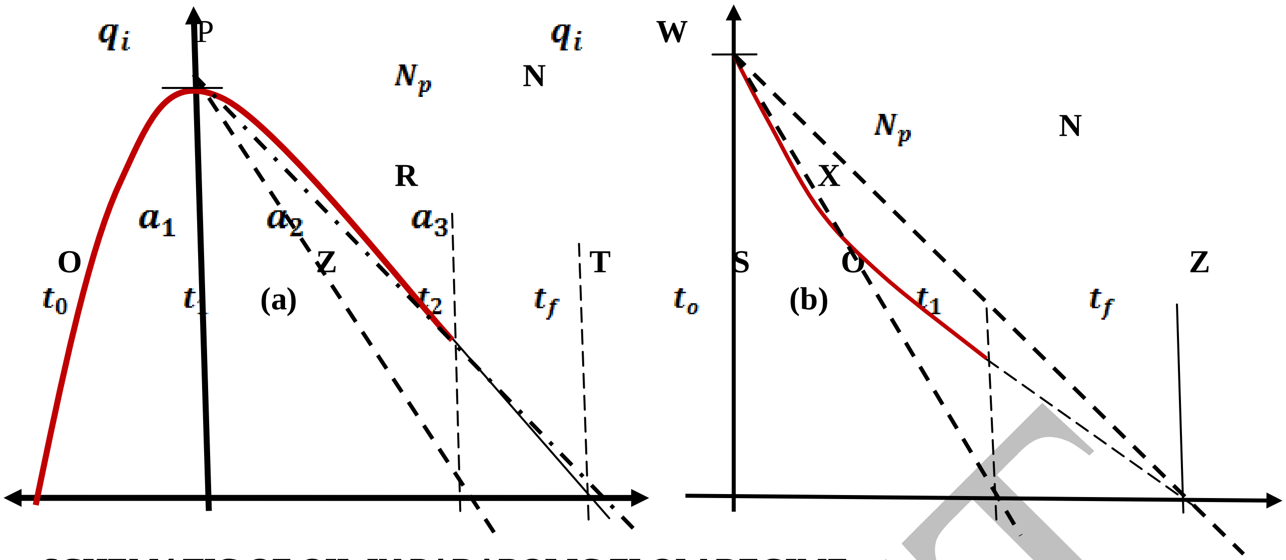


Fig 3.3

SCHEMATIC OF OIL IN PARABOLIC FLOW REGIME

EVALUATION MODEL – II

The dome shape of Fig 3.3a indicates a parabolic flow rate from lowest at point-P to a maximum point –P and declines to abandonment at point – R. The curve was extrapolated from point- R at t_1 to point–S at t_f, for estimation of gas-reserves initially in place by extension of curve-PR at point-R to S at time-t_f. In the case of Figure 3.4b the reservoir pressure is just slightly above or at the dew-point pressure. The implication of this case is that decline starts right from the early age of production at point–W to point-X, but this is not common in reservoirs. The outstanding advantages of the decline stage models include: Prediction of the daily gas production rate and cumulative recovery in a given period. This enables the operator to equally predict the abandonment period and the cumulative recovery value.

HYDROCARBONS PRODUCTION RATE DECLINE CONSTANT AND RATES MODELS:

Basically 2 types of rate decline trends were used 1st order equation where n = 1, and 2nd order equation where n = 2. The general equation for natural production of a gas field reserves is the product of the rate-constant and the actual rate raised to power-n. This is given by parabolic flow regime (Eqn3.9): Using the curve in Figure 3.3a and integrating Eqn3.9, the Governing Evaluation model was postulated. The governing equation was used to obtain hydrocarbons production rate, q and the rate-constant (b). To obtain the rate, q, remove the log and rearranging gives $q = q_i e^{-bt}$. To estimate the rate constant (b), the governing equation is applied at point-A, point-B and point-C of Figure 3.4, generating 3 equations and solving simultaneously each pair for **"b"** Egn3.12 as follows:

$$\begin{bmatrix} \text{Actual Change} \\ \text{in Production} \\ \text{Rate with Time} \end{bmatrix} = \begin{bmatrix} \text{A Decline Rate} \\ \text{Constant} \end{bmatrix} \begin{bmatrix} \text{Actual Rate in} \\ n-\text{order} \end{bmatrix}$$

$$\frac{dq}{dt} = -bq^n \qquad 3.9$$

When $n = 1$: 1^{st} ***Order Decline Rate Parabolic Flow***

$\int_{q_i}^{q} \frac{dq}{q} = -b \int_o^t dt$ or $lnq - lnq_i = -bt$ (**The Governing Equation**)

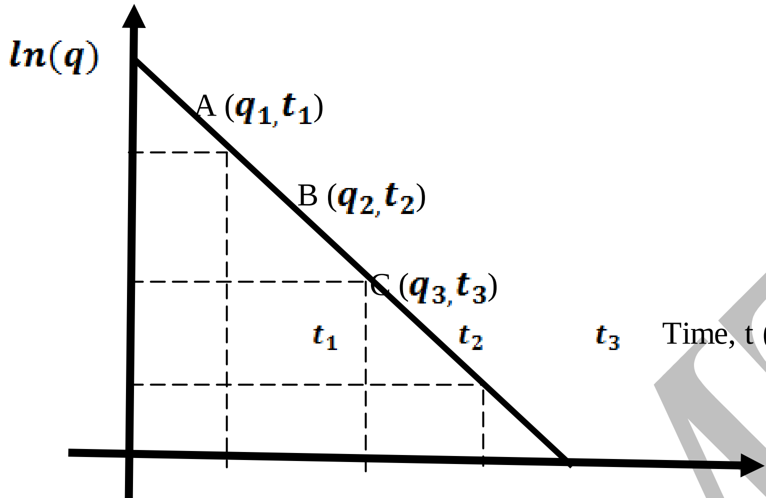


Fig 3.4 Parabolic Rate Decline Plot

$$lnq_1 - lnq_i = -bt_1$$
3.10
$$-\ (lnq_2 - lnq_i = -bt_2)$$
3.11

(3.13) –(3.14) $ln\frac{q_1}{q_2} = b(t_2 - t_1)$ or $b = \frac{ln(q_1/q_2)}{t_2 - t_1}$ *and* $q = q_i e^{-bt}$

3.12

If $b_1 = b_2 = b_3 = ... = b_n$ it implies uniform decline and $n = 1$, so the equation $b = \frac{ln(q_1/q_2)}{t_2 - t_1}$, was used to project the flow rate, q for a give time, t. Using $q = q_i e^{-bt}$: $q_1\ at\ t_1,\ q_2\ at\ t_2,\ q_3\ at\ t_3, ... q_n\ at\ t_n$

When $n = 2$: 2^{nd} ***Order Decline Rate Parabolic Flow***

Solving Eqn3.13 and Multiplying LHS by q_i and rearrange gives Equ3.14, the governing equation: To estimate the rate constant, **''b''** the governing equation is similarly applied at point-A, point-B and point-C of Figure 3.4, generating 3 equations and simultaneously each pair is solved for **''b''** or just re-arranged making b the subject of the formula ($q = q_i - bqt$). Plotting $q\ vs\ t$, the slope is $-bq$ and intercept is q_i.

$$\int_{q_i}^{q} \frac{dq}{q^2} = -b \int_o^t dt \quad \text{or} \quad \frac{1}{q_i} - \frac{1}{q} = -bt$$
3.13
$$q = \frac{q_i}{(1 + bt)} \quad \text{or} \quad q = q_i - bqt$$ (**2nd order flow governing equation)**
3.14
$$q_1 = q_i - bq_1t_1$$
3.15

$$- (q_2 = q_i - bq_2t_2)$$

3.16

(3.19) –(3.20) $q_1 - q_2 = b(q_2t_2 - q_1t_1)$ or $b = \frac{q_1 - q_2}{q_2t_2 - q_1t_1}$ *or* $b = \frac{q_i - q}{q \Delta t}$

3.17

If $b_1 = b_2 = b_3 = ... = b_n$, indicating a uniform decline rate *when*, $n = 2$, so the equation, $b = \frac{q_1 - q_2}{q_2t_2 - q_1t_1}$ *or* $\frac{q_i - q}{q\ t}$, was used to project the flow rate, q for a given time, t. That is $q_1\ at\ t_1,\ q_2\ at\ t_2,\ q_3\ at\ t_3, ... q_n\ at\ t_n$, when $n = 2$.

CUMULATIVE GAS-PRODUCED (G_p) AND RESERVES INITIALLY IN PLACE (G) MODELS:

In this case the wellbore started by building up the internal energy for some time from time, t_o to time, t_1 in figure 3.3a, because the reservoir pressure was above the dew-point pressure. It built-up from the initial stage to the transient and transition stage at point–P, but the transition stage period was too short. To this effects steady state flow (called the plateau) was not observed in the curve at time, t_1 instead rate decline stage set in with short transition stage, from time, t_1 to time, t_2, Which covered cumulative gas recovery value (G_p, **M**M scf)**.** After this the rate decline stage continued from time-t_2 to time-t_f covering the Gas-Reserves initially in place value (G, MM scf). Any recovery from time-t_2 to time-t_f covers the hydrocarbons supposed to be the residual gas of that reservoir. The depletion of the gas in that reservoir from time-t_o to **time-**t_2 was cumulative gas recovery value using of the area parabola PTZO in Fig 3.3a and from time-t_o **to time-**t_f was called gas initially in place, **using t**he equation of the area of that shape (parabola PRSO in Fig 3.3a.

Hydrocarbons Production per Unit Time (MM scf/yr) Model

$$\begin{bmatrix} Total\ Hydrocarbons \\ Production\ per\ Time \end{bmatrix} = \begin{bmatrix} Area\ of \\ Curve, a_1 \end{bmatrix} + \begin{bmatrix} Area\ of \\ Curv, a_2 \end{bmatrix}$$

$$G_p = \frac{q_i}{2}[(t_1 - t_0) + (t_2 - t_1)] \quad \text{For Gas} \qquad 3.18$$

Hydrocarbons Initially in Place, MM scf (Figure 3.3a) Models

$$\begin{bmatrix} Total\ Hydrocarbons \\ in\ Place\ initially \end{bmatrix} = \begin{bmatrix} Area\ of \\ Curve, a_1 \end{bmatrix} + \begin{bmatrix} Area\ of \\ Curv, a_2 + a_3 \end{bmatrix}$$

$$G = \frac{q_i}{2}[(t_1 - t_0) + (t_f - t_1)] \quad \text{For Gas}$$

3.19

APPLICATION OF THE MODEL EQUATIONS USING REGIONAL DATA:

Using the curve in Fig 3.1, $q_i = 100\ MMscf/d$, $t_o = 0$, $t_1 = 2years$, $t_2 = 14.45yrs$, $t_3 = 22.5yrs$ and $t_f = 25.8$ were estimated. Putting these values in Eqn3.7/Eqn3.25, the decline constant using Table 3.9 $b_{15} = ln\frac{q_{15}}{q_{16}} = b_{16} = \ln\frac{q_{16}}{q_{17}} = b_{17} = ln\frac{q_{17}}{q_{18}} = ... = b_n = \frac{q_{n-1}}{q_n} = 0.2$, the cumulative gas production (G_p) was obtained and in Eqn3.26 Gas initially in place (GIIP) was also obtained. These were comparable with Standing and Katz, (1942) MBE for volumetric gas reservoir. ***Ref*: *Appendix* − *c*: $G_p = 637.06 * 10^9 scf$ *and GIIP* = $699.7 * 10^9 scf$.**

$$G_p = \frac{365.25*100}{2}[(14.45 - 2.0) + (22.5 - 0)] \quad \text{and} \quad G = \frac{365.25*100}{2}[(14.45 - 2) + (25.8 - 0)]$$

The challenge in this case was to curve-fit the plotted figure in order to extrapolate to the initial stage.

RESULTS AND DISCUSIONS:

RESULTS: EVALUATION MODEL – 1: Figure 4.1 shows schematic view of gas cumulative production and initially in place, while Table 4.1 shows the confirmed projectile evaluation models equations for projectile gas flow.

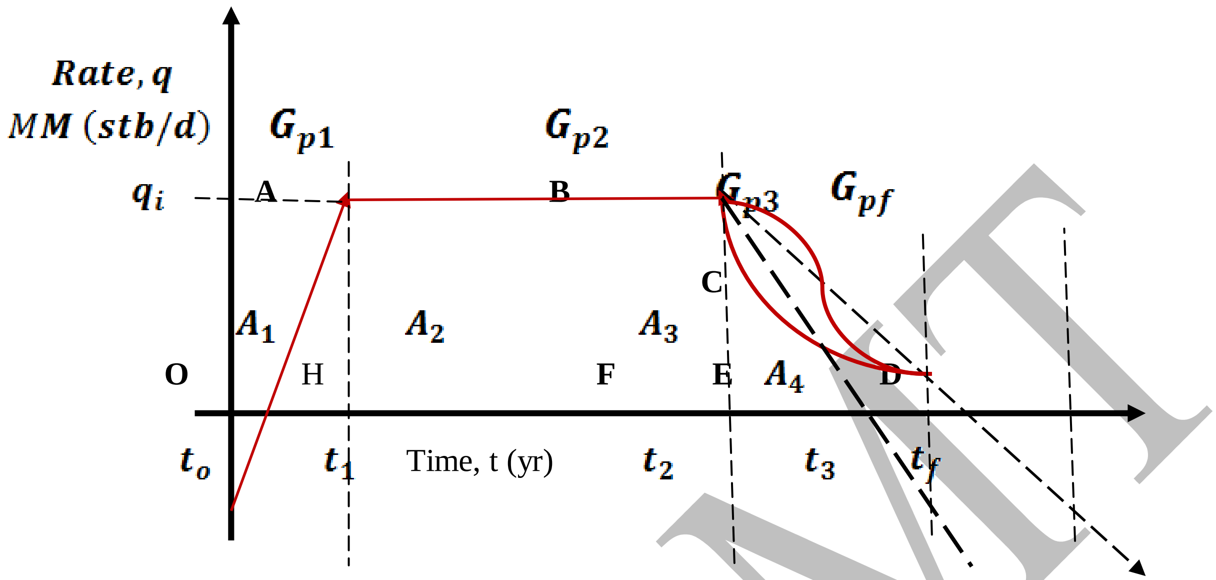


Fig 4.1 Schematic of Cumulative Production and Initial Gasl

Table 4.1: Confirmed Projectile Evaluation Models

Type	**Eqn**	**Model Equation**	**Remarks**
PROJECTILE GAS FLOW MODELS FIG 3.1	3.6 3.17 3.7	$b = \frac{ln(q_i/q)}{t-t_i}$ AND $q = q_i e^{-bt}$ $b = \frac{q_1 - q_2}{q_2 t_2 - q_1 t_1}$ AND $q = \frac{q_i}{1+bt}$ $G_p = \frac{q_i}{2}[(t_2 - t_1) - (t_3 - t_0)]$	***For*** $n = 1$ FOR $n = 2$ **CUMULATIVE GAS, MM SCF**
	3.8	$G = \frac{q_i}{2}[(t_2 - t_1) + (t_f - t_o)]$	**GAS INITIALLY IN PLACE, MM SCF**

EVALUATION MODEL–2: Figure 4.3 shows schematic view of gas cumulative production and reserves initially in place respectively, while Table 4.2 shows the confirmed parabolic fluid flow regime evaluation models equations for parabolic gas flow.

Table 4.2: Confirmed Parabolic Evaluation Models – I

Type	Eqn	Model Equations	Remarks
Fig 3.3 Decline Rate for $(n = 1)$	3.6 3.7 3.8	$q = q_i e^{-bt}$ $\quad b = \frac{ln(q_1/q_2)}{t_2 - t_1}$ $G_p = \frac{q_i}{2}[(t_1 - t_0) + (t_2 - t_1)]$ $G = \frac{q_i}{2}[(t_1 - t_0) + (t_f - t_1)]$	Per time, t Cumulative Gas, MM SCF Gas Initially in Place, MM SCF
Fig 3.3 Decline Rate for $(n = 2)$	3.17 3.18 3.19	$q = \frac{q_i}{1+bt}$ $\quad b = \frac{q_i - q}{q\,t} = \frac{q_1 - q_2}{q_2 t_2 - q_1 q_1}$ $G_p = \frac{q_i}{2}[(t_1 - t_0) + (t_2 - t_1)]$ $G = \frac{q_i}{2}[(t_1 - t_0) + (t_f - t_1)]$	Per time, t Cumulative Gas, MM SCF Gas Initially in Place, MM SCF
For Easy Unit Time Conversion	$e^{-bt} = (1-b)^t$, (Taylor''s Expansion) $(1 - b/yr) = (1 - b/m)^{12} = (1 - b/d)^{365.25}$ $b/yr = 12 * b/m = 365.25b/d$		Ref: (Taylor''s Expansion

DISCUSSIONS:

PROJECTILE DOMINATED FLUIDS FLOW REGIME:

A gas reservoir production performance naturally results into a projectile flow trend when both the internal and external energies control the flow trend. The reservoir pressure is highly above the dew point pressure. The principal mechanism which controlled the gas reservoir flow performance at the early stage was the high pressure above the dew point drive system. That was possible, because the production rate increased in the initial stage from minimum through transient part to a peak value in a given time. The peak rate value was stable for another given time called the plateau/steady. The plateau stage was equally the initial reservoir conditions before the transition stage A transition stage is a critical stage which results into a decline stage. Once the decline trend sets in, the flow rate would decline from the peak value towards the economic flow rate value called an abandonment flow rate. The decline trend is classified into two main orders the first-order and second-order. Third order equations are mainly wave propagation and are very rare, so this research work does not cover the third order equations of flow during production operations. In the third order equations, the wave tends to undergo simple harmonic motion (SHM) and most SHM tend to damped oscillation. The SHM is defined by the equation, $y = ax^n + bx + c$, with the solution as: $\mathbf{x} = \frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$. When the value of $b^2 - 4ac$ is negative, meaning that $b^2 < 4ac$ the flow equation $y = asinx + bcosx + c$ is perpetually observed, which is not common in the oil or gas fluid dynamics. Most projectile fluid dynamics or flow commonly tends to 1st order equation especially gas stream flow regimes, because of the stability in the gas stream. Another reason is that, gas dose not undergo phase changes during product operation, except in gas condensate reservoirs with much condensable components. It started by building up the internal energy for some time from time-t_o to time-t_1 in fig 4.1. The steady state flow (called the plateau) started from time, t_1 to time, t_2, after that the rate decline state set in with short or unobservable transition state, from time, t_2 to time, t_3 covering the cumulative gas recovery value (in MM scf). Any un-recovery fluid from time-t_3 to time-t_f was the hydrocarbons supposed to be the residual gas in that reservoir. The complete depletion of the hydrocarbons in that reservoir (reserves initially in place) was estimated from time-t_o to time-t_f. The equation of the area of that shape (trapezium) was used as the value of the hydrocarbons reserves initially in place. The value was confirmed with the field estimated value.

PARABOLIC FLUID FLOW REGIME:

A gas reservoir production performance naturally results into a parabolic dominated flow trend when it pressure is closed to the dew pressure. In this case the boundary conditions effects influence it after a short period of production with the boundary conditions effects right from the start of a reservoir production date. The principal flow mechanism in the gas reservoir flow performance was an internal energy drive system with some external energy effects, which decline sharply after a short time of the reservoir production. The plateau stage was absent in the parabolic dominated flow trend and the transition stage was sharp or very short. In a transition stage the flow rate tended to be unstable in another short time and the instability was not noticeable in some plots. Once the decline trend sets in just like the projectile flow, the flow rate declined from the peak value towards the economic flow rate value called an abandonment flow rate. The production rate decline trend in parabolic dominated flow trend was classified into two main orders the first-order where the decline exponent was unity ($n = 1$) and the decline trend production decline rate value 'b'' was fairly steady, second-order where the production decline rate value 'b'' was fairly steady as well, but decline exponent was two ($n = 2$). If $b_1 = b_2 = b_3 = \ldots\ldots\ldots = b_n$ and $n = 1$, it implies uniform decline so Eqn3.6 would be suitable for use in projecting the flow rate, q for a give time, t. That is $q_1\ at\ t_1,\ q_2\ at\ t_2,\ q_3\ at\ t_3, \ldots\ldots\ldots q_n\ at\ t_n$. If $b_1 = b_2 = b_3 = \ldots\ldots\ldots = b_n$ and $n = 2$, it indicates uniform decline, so Eqn3.17 would be used in projecting the flow rate, q for a give time, t. That is $q_1\ at\ t_1, .\ q_2\ at\ t_2,\ q_3\ at\ t_3, \ldots\ldots\ldots q_n\ at\ t_n$

APPLICATION OF THE EVALUATION MODELS USING GENERIC DATA:

The advantage in using generic data is mainly to enhance hydrocarbons production projected values. This makes it easy to predict future hydrocarbons production performances and take decision on the reservoir pressure management. The results showed high accuracy on the forecast. The percentage accuracy for gas fields ranged from 99.86% and above. Table 4.3 shows the comparison of the model results with the tanks, tabulated and Craze - Buckley MBE estimated values.

Table 4.3: The Model Results for Gas Compared with the Tank and MBE Values

S/No	Value Used	$G_p, MMscf$	Accuracy	$G, MMscf$	Accuracy
1.	Models	637.40		698.54	
2.	Tanks	637.06	99.86%	-	99.98%
3.	Tabulated Tables	639.20		-	
4.	Craze and Buckley MBE	639.20		699.7	

CONCLUSION AND RECOMMENDATIONS:

CONCLUSION:

Mathematical models equations were successfully derived for studying reservoirs fluids depletion from the peak value at decline stage to an economic value called abandonment. Decline rate trends analysis showed two types of flow projectile dominated flow regimes attributed to high pressure above the dew point and boundary conditions effects while Parabolic flow regimes whose principal mechanism is due to internal and boundary conditions effects. The projectile dominated flow models were mainly used and generated curves for predicting hydrocarbons production performances. When the reservoir pressure is above its dew-point pressure, projectile dominated flow is possible and evaluation models-I should be used, but when the reservoir pressure is closed or at dew-point pressure, the parabolic dominated flow is possible in that well and evaluation models-II should be used. This is because the dew-point pressure is the critical point for critical rate. Highly above the dew- point the dominated fluids flow is the projectile type, while slightly above or at the dew-point pressure down to the abandonment point parabolic dominated fluid flow regime is expected. The parabolic dominated fluid flow models were used to predict future recovery from the decline stage to an economic rate (abandonment). The extrapolation of the curve from the decline point to the economic rate point on the t-axis at t_f gave the total reserves in place.

OBSERVATIONS:

On like oil gas reservoirs do not undergo phase changes during production, so both projectile and parabolic flow trend rates declines are fairly uniform. The only condition that might change this principle is when external energy influences it otherwise. When yearly rates projected to abandonment or close to it are used to generate curves, the models give high accuracy estimated reserves (N_p and N). When first and last points of the decline stages are extrapolated for actual flow rate (q) and time (t), as the models input data, they give high accuracy estimated reserves (N_p and N). **Yearly r**ates and pressure depletion trend synergy was necessary to predict transient and steady states periods, but was not used here. Projected production performances of reserves and estimation of the reserves initially in place percentage accuracy for gas fields ranged from 99.86% and above.

RECOMMENDATIONS:

a. First and last points of the decline stage must be extrapolated to the axes in order to obtain actual flow rate, q and time-t as the model input, for high estimation accuracy.
b. Only projected rates to abandonment stage or close to it should be used to estimate reserves. It improves reserves estimation accuracy.
c. Yearly rates and pressure decline synergy is not used and production depends on pressure sustainability. Hence it is recommended that pressure maintenance should be used (if required) to manage the reservoir pressure for economic recovery

NOMENCLATURE:

A: Area of the reservoir, $acres\ or\ ft^2$
a : Actual decline fraction of production rate
a_1 : = Initial oil or gas production decline
Agbada Formation: Geological formation which consists mainly of sandstones, shale alternation with the sandstones predomination
Akata Formation: This is a Marine pro-Delta mainly shale-stones and siltstones, which crop out in sub-sea outer Delta.
AGA: American Gas Association, generally acceptable standard units
^{o}API: Oil or Gas Gravity, API (American Petroleum Institute)
b: Rate Decline Constant, $Mbbl/yr\ or\ Mbbl/d$
Bbl: Barrel (Unit of oil or liquid measurement)
Benin formations: This is mainly sand and sandstones, coarse to fine, granular in texture and partly unconsolidated formation.
B_{of}: Actual oil formation volume factor, rb/stb
B_{oi}: Initial Oil formation volume factor, rb/stb
Bubble Point Pressure: Critical pressure condition for rate decline initiation
CAPEX: Capital Expenses (Development Bills)
D: Depth of the reservoir, ft
DCA (Decline Curve Analysis): Mathematical equations, tabulated tables or graphical procedures for studying the oil and/or gas production rate, prediction of cumulative oil or projected oil production
Decline Curve (Tend): Graphical representation of oil or gas production rate
Decline Rate: Reduction of a production volume per unit time, $Mbbl/yr$
DPR: Department of Petroleum Resources, NNPC subsidiary

DFCA: Dynamic Fluids Computational Analysis
E_i: Gas expansion factor, %
G: Gas initially in place (GIIP), MMscf
G_p: Cumulative gas recovery in a reservoir, MMscf
GOR: Gas - Oil Ratio, scf/bbl
MBE: Materials Balance Equation (quick volume changes estimation)
n: Rate Decline exponential or production decline rate power constants
N: Oil initially in place (OIIP), stb
NNPC: Nigerian National Petroleum Cooperation (Oil / Gas operation Age)
N_P: Cumulative oil production in a reservoir, stb
OPEX: Operations Expenses (Daily operation costs or bills)
q: Actual hydrocarbons flow rate, bbl/d or bbl/yr
q_i : Initial oil or gas production flow rate, bbl/d or bbl/yr
STB or stb: stalk tank barrel
SCF or scf: Standard cubic feet, ft^3
t : Time unit (s, hr or yr)
Transient Part or Stage: Unsteady rate in the initial stage of production
Transition Stage: A critical stage which could result into a decline stage
$t_o, t_1, \dots \dots \dots . t_f$: Unit time, ft
γ_g: Gas specific gravity, dimensionless
Z: Gas deviation or compressibility factor, %8
$\frac{1}{n}$ = h = hyperbolic decline constant

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